	Policy Name Calculation policy		Date Adopted: Autumn 2017 By whom: Standards and Performance
Date for review: Spring 2018	**Applicable to:** Whole school	**Staff member responsible:** Alex Bingham	**Governor committee responsible:** Standards and Performance

Introduction:

Children are introduced to the processes of calculation through practical, visual, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both **conceptual understanding** and **fluency** in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use models and images to support their mental and written methods of calculation. This year (2017) are beginning to embed the use of the bar model to support children's understanding of calculations and maths problems. As children's mental methods are strengthened and refined they begin to work more efficiently, which will support them with using succinct written calculations. At Middle Barton School we use the Concrete – Pictorial – Abstract approach (CPA) to build on conceptual and procedural understanding. Concrete and pictorial representations of mathematics are chosen carefully to help build procedural and conceptual knowledge together. Exercises are structured with great care to build deep conceptual knowledge alongside developing procedural fluency.
The focus is on the development of deep structural knowledge and the ability to make connections. Making connections in mathematics deepens knowledge of concepts and procedures, ensures what is learnt is sustained over time, and cuts down the time required to assimilate later concepts and techniques.

By the end of Year 6, children will be equipped with efficient mental and written calculation methods, which they use with fluency. Decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. At whatever stage in their learning, and whatever method is being used, children's strategies must still be underpinned by a secure understanding and knowledge of number facts that can be recalled fluently. The overall aims are that when children leave primary school they:

- Are able to recall number facts with fluency, having developed conceptual understanding through being able to visualise key ideas – such as those related to place value - through experience with practical equipment and visual representations;
- Make use of diagrams and informal notes to help record steps and part answers when using mental methods that generate more information than can be kept in their heads;
- Have an efficient, reliable, written method of calculation for each number operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;
- Are able to make connections between all four number operations, understanding how they relate to one another, as well as how the rules and laws of arithmetic can be applied.

This Calculation policy outlines the strategies for teaching in each year group and gives examples of how the CPA approach can support the teaching and learning of calculations in each year group. This will ensure progression, consistency and in depth understanding throughout the school.

Monitoring
This policy will be monitored by the maths leader through book scrutiny, lessons observations and learning journeys throughout the year in line with the Senior Leadership Team's monitoring schedule. Staff will receive feedback on these actions following each round of monitoring in order to best develop their practice.

Addition

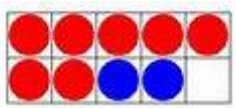
Foundation Stage

Addition as 'combining 2 groups'

add and subtract two single-digit numbers

Practical / recorded using ICT (eg digital photos / pictures on IWB)

Pictorial: Tens Frames

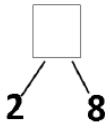


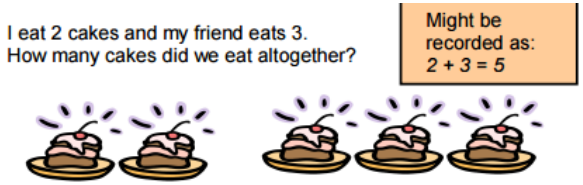
Numbered number tracks/lines

Grouped (circled) objects

Pictures representing a story

Cherry Model





I eat 2 cakes and my friend eats 3. How many cakes did we eat altogether? Symbols 8 people are on the bus. 5 more get on at the next stop. How many people are on the bus now?

Concrete resources to support conceptual understanding: **Numicon**

Manipulatives such as compare bears, unifix cubes, counters

Place value counters

Bead Strings

Dominoes

Dice

Bar Model

5	
3	2

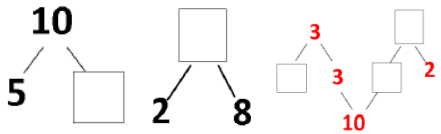
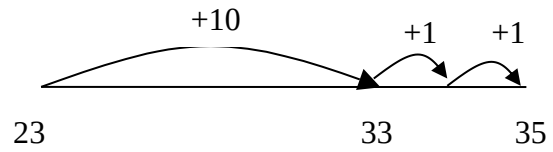
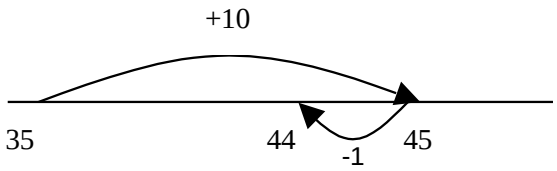
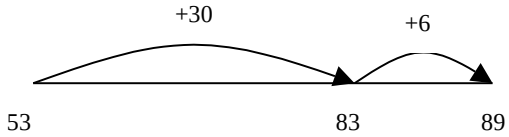
Abstract recorded as: 2 + 3 = 5

Fluency: one more, counting forwards and backwards in ones to 20

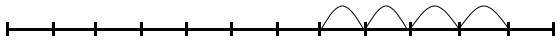
- Recognises numerals 1 to 5.
- Recognise some numerals of personal significance
- Counts up to three or four objects by saying one number name for each item.
- Counts actions or objects which cannot be moved.
- Counts objects to 10, and beginning to count beyond 10.
- Counts out up to six objects from a larger group.
- Selects the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Counts an irregular arrangement of up to ten objects.
- Estimates how many objects they can see and checks by counting them.
- Uses the language of 'more' and 'fewer' to compare two sets of objects.
- Finds the total number of items in two groups by counting all of them.
- Says the number that is one more than a given number.

Early learning goal – numbers
Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

- Finds one more or one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.
- Records, using marks that they can interpret and explain.
- Begins to identify own mathematical problems based on own interests and fascinations.

Year One	Year Two	Year Three
Addition as 'counting on' 1 digit + 1 digit (bridging 10) 2 digit + 1 digit (bridging 20) **Pictorial** Use pictures and objects  X X + X = 5 **Part / Part whole**  **Bar Model** 10 / 5 / 5 10 / 2 / ? / ? **Number Lines** Using number lines to teach (Teacher model number lines with missing numbers to deepen learning)	**2 digit + 2 digit (bridging 10s)** **Pictorial: partition into tens and ones and recombine using number line** 12 + 23 = 10 + 2 + 20 + 3 = 30 + 5 = 35 **refine to partitioning the second number only:** 23 + 12 = 23 + 10 +1 + 1 = 33 + 1 + 1 = 35  *Mental Method* **Add 9 or 11 by adding 10 and adjusting by 1** 35 + 9 = 44  	**2 digit + 2 digit (bridging 100) 3 digit + 2 digit (not bridging 1000) 3 digit + 3 digit (not bridging 1000)** **Pictorial: partition into tens and ones and recombine** Partition both numbers and recombine. Refine to partitioning the second number only e.g. 36 + 53 = 53 + 30 + 6 = 83 + 6 = 89  **Add a near multiple of 10 to a two-digit number** Continue as in Year 2 but with appropriate numbers e.g. 35 + 19 is the same as 35 + 20 – 1. **Partition into hundreds, tens and ones and recombine** Either partition both numbers and recombine or partition the second number only e.g. 358 + 73 = 358 + 70 + 3 = 428 + 3 = 431

7 + 4 = 11



Concrete resources to support conceptual understanding: beadstrings, counters, unifix and Numicon

Children are encouraged to be familiar with the colours and shapes of the individual pieces so that they can use them to calculate without the need for counting.

Children can identify the shape of 2 combined shapes and use a 10 piece to 'see' teen numbers.

Doubles and halves can easily be

demonstrated.

+ = signs and missing numbers

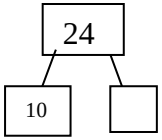
3 + 4 = □ □ = 3 + 4
3 + □ = 7 7 = □ + 4
□ + 4 = 7 7 = 3 + □
□ + ∇ = 7 7 = □ + ∇
? + ? + ? = 7

Promoting covering up of operations and numbers.

Using mistakes / Misconceptions

Some pupils might count the number that they are starting from as one of the numbers in the count when adding on or counting back.

Part / Part whole



Bar Model

30	
20	10

30		
5	?	?

Concrete resources to support conceptual understanding:: Numicon/Diennes/Place Value Counters

+ = signs and missing numbers

Continue using a range of equations as in Year 1 but with appropriate, larger numbers.

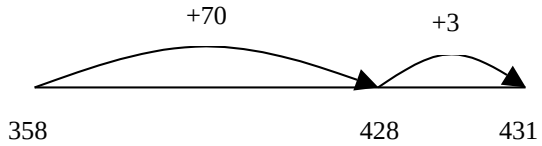
Extend to

14 + 5 = 10 + □

and adding three numbers

32 + □ + □ = 100 35 = 1 + □ + 5

Column Addition

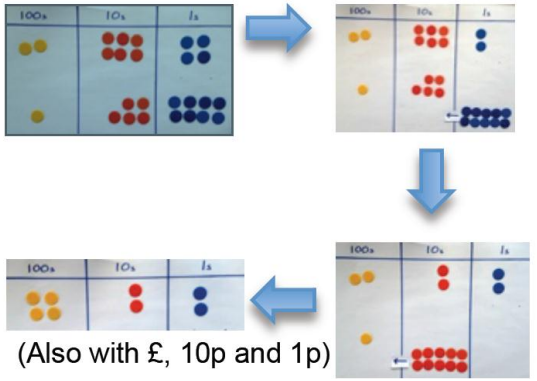


Bar Model

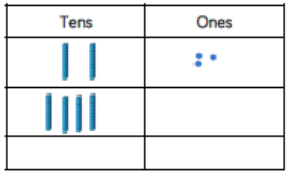
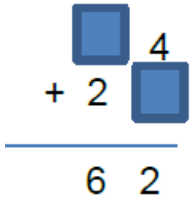
48	
10	38

48		
10	?	?

Concrete resources to support conceptual understanding:: Numicon/Diennes/Place Value Counters



Column Addition

Some pupils may appear to be counting confidently but they may just be mimicking the rhythm of the counting pattern.	Introducing the column addition method to the children alongside pictorial and concrete objects. Use the place value charts and concrete materials to complete the calculations.  2 3 + 4 0 _____ **Using mistakes / Misconceptions** • Some pupils may include the first number in the count (not count on from) • Some pupils may confuse the language of addition or subtraction, and therefore use the incorrect operation to carry out a calculation Use sum to refer to calculations that involve addition only.	The expanded methods is the first step into the formal written method, followed by the compact method. Use Concrete resources to support conceptual understanding: diennes, place value. Write the '1' underneath. 100+30+4 200+30+3 300+60+7 134 +233 367 +245 163 408 1 **+ = signs and missing numbers**  **Using mistakes / Misconceptions** • Some pupils may carry the wrong carry digit (i.e. the ones digit rather than the tens digit)
Fluency: Number bonds: 5, 6 Largest number first. Number bonds: 7, 8 Add 10. Number bonds: 9, 10 Ten plus ones. Doubles up to 10 Use number bonds of 10 to derive bonds of 11	**Fluency: 10 more Number bonds: 20, 12, 13 Number bonds: 14,15 Add 1 digit to 2 digit by bridging. Partition second number, add tens then ones Add 10 and multiples. Number bonds: 16 and 17 Doubles up to 20 and multiples of 5 Add near multiples of 10. Number bonds: 18, 19 Partition and recombine**	**Fluency: Add multiples of 10, 100 Add single digit bridging through boundaries Partition second number to add pairs of 100 Use near doubles to ad Add near multiples of 10 and 100 by rounding and adjusting Partition and recombine**
Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7= – 9	**Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: * a two-digit number and ones * a two-digit number and tens * two two-digit numbers * adding three one-digit numbers**	**Add and subtract numbers mentally, including: * a three-digit number and ones * a three-digit number and tens * a three-digit number and hundreds**

Addition

Year Four

3 digit + 2/3 digit (incl bridging 1000) Decimals: money (£7.85 + £3.49)

Formal Written Methods

Compact method writing the '1' underneath.

3456
2346 +
5802
11

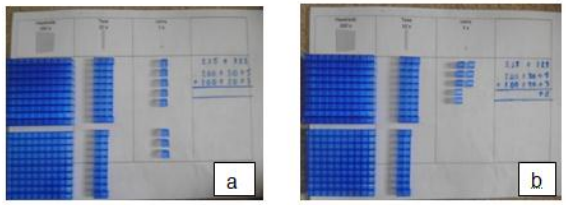
Bar Model

2000	
1200	800

2000		
1200	?	?

Concrete resources to support conceptual understanding: diennes/place value counters

Illustration of how to use Dienes equipment to ensure children



Year Five

4 digit + 3 digit. Decimals up to 2dp (23.7 + 48.56)

Formal Written Methods

Numbers with at least four digits
3587 + 675 = 4262

3587
+ 675
4262
1 1 1

Bar Model

3456	
2020	1436

3456		
2300	?	?

Concrete resources to support conceptual understanding: diennes/place value counters

+ = signs and missing numbers

6 □ 0 2 □
\+ 5 □ 5 1
□ 9 1 8 0

Mistakes and Misconceptions

To avoid confusion with language, all teachers use 'sum' to refer only to the result of an addition.

Year Six

Consolidate / extend Y5 including: Three numbers Decimals up to 3dp (context: measures)

Formal Written Method

Extend to numbers with any number of digits and decimals with 1 and 2 decimal places.
124.9 + 117.25 = 242.15

124.90 ***add in a zero to keep the place value***
+ 117.25
242.15
1 1

Bar Model

117.3	
76.5	40.8

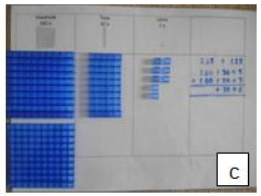
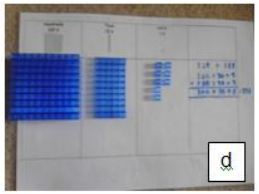
117.3		
?	?	?

Missing number calculations

3210 + 2564 = 9836 - _____
2678 + _____ = 9305 – 3789

Mistakes and Misconceptions throughout previous years.

To avoid confusion with language, all teachers use 'sum' to refer only to the result

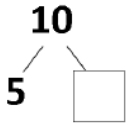
have an understanding of place value when using columnar addition.   **+ = signs and missing numbers** 432 + ☐ = 770 50 + 199 + ☐ = 450 **Mistakes and Misconceptions** *To avoid confusion with language, all teachers use 'sum' to refer only to the result of an addition. Teachers say 'complete these calculations' instead of 'complete these sums' When carrying, those numbers being carried are placed beneath the answer line.* *Using place value headings incorrectly.*	*Teachers say 'complete these calculations' instead of 'complete these sums'* *When carrying, those numbers being carried are placed beneath the answer line.*	*of an addition. Teachers say 'complete these calculations' instead of 'complete these sums'* *When carrying, those numbers being carried are placed beneath the answer line.*
Fluency: Add multiples of 10s , 100s, 1000s Fluency of 2 digit + 2 digit Partition second number to add Decimal pairs of 10 and 1 Use near doubles to add Adjust both numbers before adding Add near multiples Partition and recombine	**Fluency: Add multiples of 10s , 100s, 1000s, tenths, Fluency of 2 digit + 2 digit including with decimals Partition second number to add Use number facts, bridging and place value Adjust numbers to add Partition and recombine**	**Fluency: Add multiples of 10s , 100s, 1000s, tenths, hundredths Fluency of 2 digit + 2 digit including with decimals Partition second number to add Use number facts, bridging and place value Adjust numbers to add Partition and recombine**
Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why	**Add and subtract numbers mentally with increasingly large numbers**	**Perform mental calculations, including with mixed operations and large numbers**

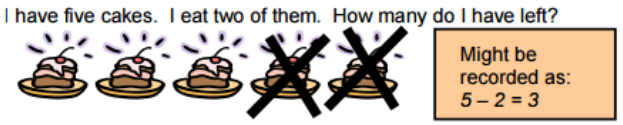
Subtraction

Foundation Stage

Subtraction as 'taking away' from a group Practical or recorded using ICT (eg digital photos / pictures on IWB)
add and subtract two single-digit numbers
Practical or recorded using ICT (eg digital photos / pictures on IWB)

Pictorial I have five cakes. I eat two of them. How many do I have left?
Tens Frames
Numbered number line/tracks
Grouped (circled) objects
Pictures representing a story
Tens Frames
Cherry Model





Concrete resources to support conceptual understanding: **Numicon**
Manipulatives such as compare bears, unifix cubes, counters
Place value counters
Bead Strings
Dominoes
Dice

Bar model

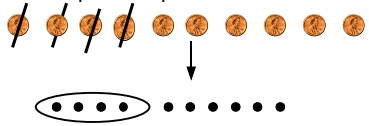
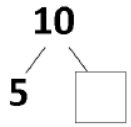
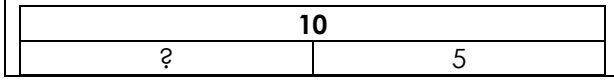
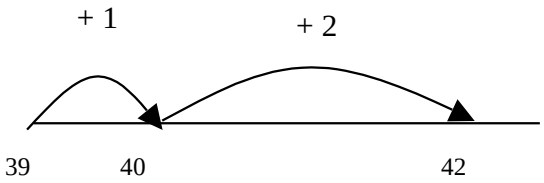
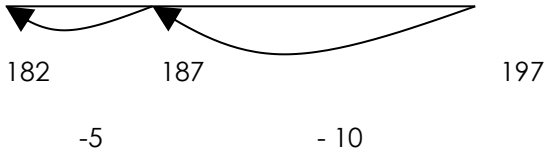
10	
?	5

Abstract
10 – 5 = 5

Fluency: one less, counting backwards from 20 in ones

- Recognises numerals 1 to 5.
- Recognise some numerals of personal significance
- Counts up to three or four objects by saying one number name for each item.
- Counts actions or objects which cannot be moved.
- Counts objects to 10, and beginning to count beyond 10.
- Counts out up to six objects from a larger group.
- Selects the correct numeral to represent 1 to 5, then 1 to 10 objects.
- Counts an irregular arrangement of up to ten objects.
- Estimates how many objects they can see and checks by counting them.
- Uses the language of 'more' and 'fewer' to compare two sets of objects.
- Finds the total number of items in two groups by counting all of them.
- Says the number that is one more than a given number.
- Finds one more or one less from a group of up to five objects, then ten objects.
- In practical activities and discussion, beginning to use the vocabulary involved in adding and subtracting.
- Records, using marks that they can interpret and explain.
- Begins to identify own mathematical problems based on own interests and fascinations.

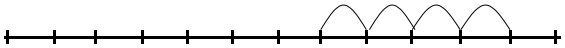
Early learning goal – numbers
Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Year One	Year Two	Year Three
Subtraction as 'taking away' and 'difference' (by counting on) 1 digit – 1 digit and 2 digit – 1 digit (bridging 10) **Pictorial** Sam spent 4p. What was his change from 10p?  **Part / Part whole**  **Bar Model**  	**Subtraction as inverse of addition 2 digit – 2 digit (bridging 10s)** **Pictorial: Number Line** 42 – 39 = 3  **Mental Method** Subtract 9 or 11. Begin to add/subtract 19 or 21 35 – 9 = 26	**Find a small difference by counting up** with appropriate numbers e.g. 102 – 97 = 5 **Use known number facts and place value to subtract** 3 digit number – 2/3 digit number. 197 – 15 = 182  Complementary addition 84 – 56 = 28

10	
2	?

Number lines

The difference between 7 and 11
(Counting on)
To reinforce concept. Practical strategies essential to see 'difference'.



- = signs and missing numbers

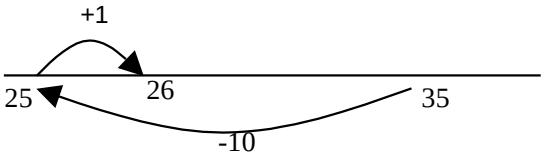
7 - 3 = □ □ = 7 - 3
7 - □ = 4 4 = □ - 3
□ - 3 = 4 4 = 7 - □
□ - ∇ = 4 4 = □ - ∇

Numicon

This continues to be used to help children visualise numbers and the can be drawn around for early recording. Children will be encouraged to use numerals to represent the numbers they are subtracting to write number sentences.

Mistakes and Misconceptions

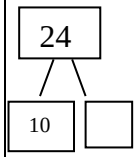
When exploring related number facts some pupils may just manipulate the numbers and record incorrectly. For example instead of understanding that they need to subtract from the largest number (for example 10 – 6 =4) they



Use known number facts and place value to subtract (partition second number only)

37 – 12 = 37 – 10 – 2
= 27 – 2
= 25

Part / Part whole



Bar Model

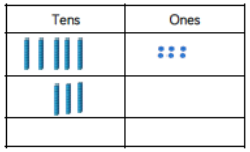
34	
?	15

24	
10	?

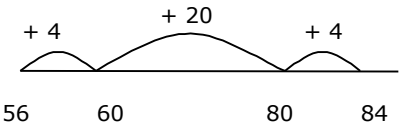
- = signs and missing numbers

Continue using a range of equations as in Level 1 but with appropriate numbers.
Extend to 14 + 5 = 20 - □
Find a small difference by counting up

Column subtraction



5 6
– 3 0
_____



Formal written methods

Numbers without, then with exchanging. Expanded method as a first step.

274 - 219 = 55

60 1
- 200 + 70 + 4
200 + 10 + 9
0 + 50 + 5

6 1
- 2 7 4
2 1 9
5 5

Bar Model

134	
?	15

124	
104	?

- = signs and missing numbers

33?4
144?
1?42

Concrete resources to support conceptual understanding: diennes/place value counters

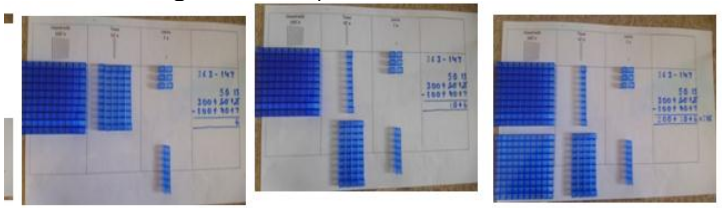


Illustration of how to use Dienes equipment to ensure children understand transference of numbers when using columnar subtraction.

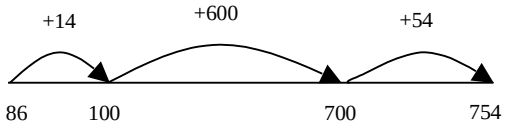
may move the numbers around because they think they are related (for example 6 – 4 = 10).	Introduce the column method to the children alongside the use of concrete and pictorial resources to reinforce understanding. **Concrete resources** to support conceptual understanding: Numicon/diennes This continues to be used to help children visualise numbers and the can be drawn around for early recording. Children will be encouraged to use numerals to represent the numbers they are subtracting to write number sentences.  **Mistakes and Misconceptions** Some children may assume commutativity within subtraction and say '2 take away 7' when they should say '7 take away 2'. Many children may think that 2 take away 7 is not possible. It is possible (when negative numbers are introduced in Stage 4), and care with language now will lessen problems with misconceptions later.	**Mistakes and Misconceptions** • Some pupils incorrectly assume and use commutativity within column subtraction; for example: 9 2 6 – 7 3 4 2 1 2 Pupils may not apply understanding of place value. Some pupils may add instead of subtract.
Fluency: Number bonds, subtraction: 5, 6 Count back Number bonds, subtraction: 7, 8 Subtract 10. Number bonds, subtraction: 9, 10 Teens subtract 10. Difference between	**Fluency: 10 less Number bonds, subtraction: 20, 12, 13 Number bonds, subtraction: 14, 15 Subtract 1 digit from 2 digit by bridging Partition second number, count back in 10s then 1s Subtract 10 and multiples of 10 Number bonds, subtraction: 16, 17 Subtract near multiples of 10 Difference between Number bonds, subtraction: 18, 19**	**Fluency: Subtract multiples of 10 and 100 Subtract single digit by bridging through boundaries Partition second number to subtract Difference between Subtract near multiples of 10 and 100 by rounding and adjusting Difference between**
Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7= Pictures / –9	**Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: * a 2-digit number and ones * a 2-digit number and tens * 2 2-digit numbers * adding 3 1-digit number**	**Add and subtract numbers mentally, including: * a 3-digit number and ones * a 3-digit number and tens * a 3-digit number and hundreds**

Subtraction

Year Four

3 digit – 2 digit 3 digit – 3 digit
And with decimals: money (£7.85 - £3.49)

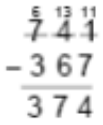
Pictorial: Number line
Counting forwards
754 – 86 = 668



Formal Written Methods: expanded and compact, carrying tens.

700 20 3
400 50 8

600 110 13
400 50 8
200 60 5



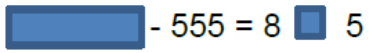
Also with exchanging numbers.

Bar Model

4030	
?	1029

3245	
1045	?

Missing numbers

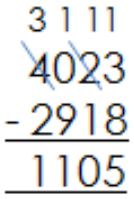


Year Five

5 digit – 5 digit
Subtraction of decimals up to 2dp (72.5 – 45.7)

Formal Written Method

Subtraction with at least four digits – same method as year 4.



6.1 – 0.9 = 5.2

Bar Model

8001	
?	1001

32000	
12500	?

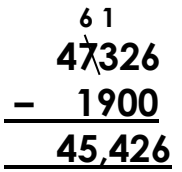
Missing Numbers

Year Six

Consolidate / extend Y5 including:
Decimal to 3 dp relating to measures

Formal Written Method

Columnar subtraction using up to 6 digits and subtraction of decimals from whole numbers and decimal numbers.



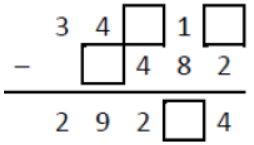
12 – 6.01 =

Bar Model

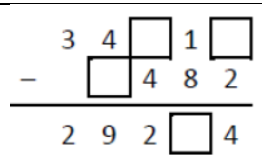
117.3	
76.5	?

117.3	
?	101.3

Missing Numbers



Mistakes / Misconceptions

Mistakes / Misconceptions • When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166. • Some pupils incorrectly assume and use commutativity within column subtraction; for example: 7 4 1 2 6 – 2 3 7 3 4 5 1 6 1 2 • Some pupils may not use place value settings correctly (especially when the numbers have a different number of digits)	 **Mistakes / Misconceptions** • When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166. • Some pupils incorrectly assume and use commutativity within column subtraction; for example: 7 4 1 2 6 – 2 3 7 3 4 5 1 6 1 2 • Some pupils may not use place value settings correctly (especially when the numbers have a different number of digits)	Some pupils may write statements such as 140 - 190 = 50 When subtracting mentally some pupils may deal with columns separately and not combine correctly; e.g. 180 – 24: 180 – 20 = 160. Taking away 4 will leave 6. So the answer is 166.
Fluency: Subtract multiples of 10s , 100s, 1000s,tenths, Fluency of 2 digit - 2 digit including with decimals Partition second number to subtract Difference between Adjust numbers to subtract Difference between	**Fluency: Subtract multiples of 10s , 100s, 1000s,tenths, Fluency of 2 digit - 2 digit including with decimals Partition second number to subtract Difference between Adjust numbers to subtract Difference between**	**Fluency: Subtract multiples of 10s , 100s, 1000s, tenths, hundredths Fluency of 2 digit - 2 digit including with decimals Partition second number to subtract Use number facts bridging and place value Adjust numbers to subtract Difference between**
Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why	**Add and subtract numbers mentally with increasingly large numbers**	**Perform mental calculations, including with mixed operations and large numbers**

Multiplication

Foundation Stage

To begin to understand the vocabulary of 'grouping' and 'lots of' doubling

Count repeated groups of the same size (1s / 2s / 5s / 10s)

Pictorial
Share out objects into groups of equal amounts
There are 3 sweets in one bag.
How many sweets are there in 3 bags?

Concrete resources to support conceptual understanding: **Numicon**
Manipulatives such as compare bears, unifix cubes, counters
Place value counters
Bead Strings
Dominoes
Dice
Numbertracks/Cuisenaire

Bar Model

9		
3	3	3

Abstract 3 + 3 + 3 = 9

Fluency: Counting on in 1s and 2s

Early learning goal – numbers

Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Year One

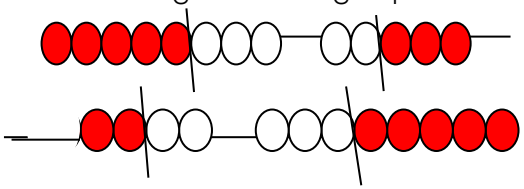
Solve (practical) problems that involve combining groups of 2, 5 or 10

Pictorial

There are 3 sweets in one bag.
How many sweets are there in 5 bags?

(Recording on a number line modelled by the teacher when solving problems)

Use of bead strings to model groups of.



Bar Model

10				
2	2	2	2	2

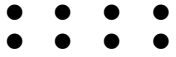
?		
5	5	5

Abstract **3 + 3 + 3 + 3 + 3 = 15**

Year Two

Multiplication as repeated addition and arrays

Arrays and repeated addition

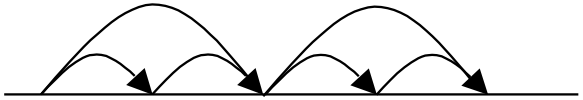


4 x 2 or 4 + 4

2 x 4

or repeated addition

2 + 2 + 2 + 2



0 1 2 3 4 5 6 7 8

Doubling multiples of 5 up to 50

15 x 2 = 30

Partition

(10 x 2) + (5 x 2)
20 + 10

= 30

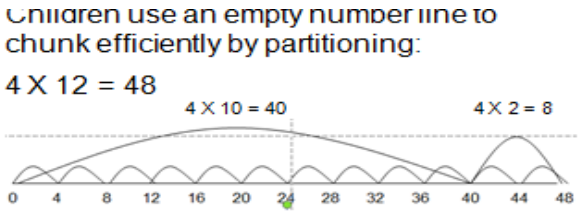
Bar Model

15				
3	3	3	3	3

Year Three

2 digit x 1 digit (e.g. 13 x 4)

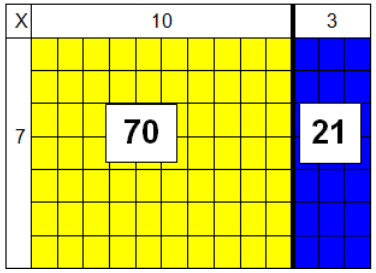
Chunking on a number line/empty number line



or partitioning

24 X 2
4 X 2 = 8
20 X 2 = 40
40 + 8 = 48

Formal Method 2 digit multiplied by 1 digit numbers using a grid method



Numicon can be used to reinforce the repeated addition and can be recorded by drawing around or printing the numicon.

Signs and Missing numbers

2 x 5 = ?
? = 2 x 5
2 x ? = 10
5 x ? = 10

Mistakes / Misconceptions

- Some pupils may appear to be counting confidently but they may just be mimicking the rhythm of the counting pattern.
- Some pupils may not be confident in counting over the tens boundaries
- Some pupils may not understand that a number can be used to show/ label the final (cardinal) value of the set of objects being counted. If a pupil is asked to count a group of objects and then asked how many there are in the set, if they have to count again, then they do not have an understanding that the final number that they have said represents the value of the objects in the set.
- Some pupils may muddle the 'teen' and the 'ty' numbers
    - Some pupils may read the units digit before the tens digit.

10		10	
5	5	5	?

Abstract 2 x 4 = 8

Concrete resources to support conceptual understanding: Numicon/Number tracks and cuisenaire
This can be used to reinforce the repeated addition and can be recorded by drawing around or printing the numicon.

x = signs and missing numbers

7 x 2 = □	□ = 2 x 7
7 x □ = 14	14 = □ x 7
□ x 2 = 14	14 = 2 x □
□ x ∇ = 14	14 = □ x ∇

Mistakes / Misconceptions

- Pupils may think that all numbers ending in 3 are multiples of 3
- Pupils may not spot patterns because they are not secure in the stable order of our number system

Bar Model

80			
20	20	20	20

8			
2	2	2	2

Abstract 2 x 8 =

x = signs and missing numbers

Concrete resources to support conceptual understanding: Numicon/place value counters on grids and Number tracks and cuisennaire

Mistakes / Misconceptions

- Some pupils 'see' the times tables as a list of 12 unconnected facts
- Some pupils do not understand multiplication is commutative.

Fluency: Count in 2s Count in 10s **Doubles up to 10** **Count in 5s** **Double multiples of 10** **Count in 2s, 5s and 10s**	**<u>Fluency:</u> 2 x table 10 x table** **Doubles up to 20 and multiples of 5** **5 x table** **Count in 3s 2 x, 5 x and 10 x tables**	**Fluency: Review 2x, 5x and 10x** **4x table 8 x table 3 x table 6 x table** **Double two digit numbers**
Solve one---step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	**Show that multiplication of two numbers can be done in any order(commutative) and division of one number by another cannot Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts**	**Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two digit numbers times one-digit numbers, using mental methods**

Multiplication		
Year Four	**Year Five**	**Year Six**
Record, support and explain: 2 digit x 1 digit (eg 16 x 8; 43 x 6) **<u>Formal Methods</u>** Expanded method followed by compact method 2 digit multiplied by 1 digit 76 X 4 24 280 304 1 4 x 6 = 24 4 x 70 = 280 280 + 24 = 304 **<u>Using Factor Pairs</u>** 13 x 12 can be solved by using factor pairs e.g 13 x 3 x 4 or 13 x 2 x 6. **<u>Bar Model</u>** **54**	**Refine and use efficient methods: 3 digit x 1 digit, 2 digit x 3 digit, digit with 1 dp x 1 digit** **<u>Formal Written Methods</u>** Build up to Short multiplication 2 digit multiplied by 4 digit number. 346 X 27 42 280 2100 120 800 6000	**Use efficient methods: 4digit x 1/2 digit (eg 2307 x 8(2)) Decimal x 1 digit (eg 31.6 x 7)1 digit with 2 dp x 1 digit number** **<u>Formal Written Methods</u>** 7.2 X 3 21.6 Long multiplication: **Long multiplication** 24 × 16 becomes Answer: 384 124 × 26 becomes Answer: 3224 124 × 26 becomes Answer: 3224

6	6	6	6	6	6	6	6	6

44			
11	11	11	11

Concrete resources to support conceptual understanding: place value counters on grids and Number tracks and cuisennaire

Signs and Missing Numbers

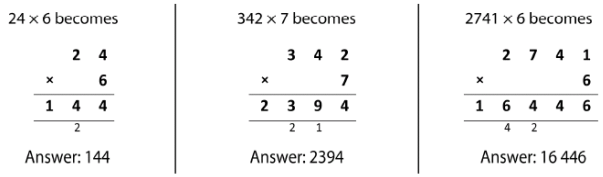
47?
× 3
?422

Mistakes / Misconceptions

- Some pupils may carry the wrong digit when using short multiplication

9342

Short multiplication



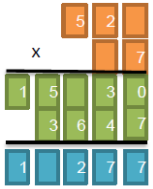
Bar Model

81								
9	9	9	9	9	9	9	9	9

48			
12	12	12	12

Concrete resources to support conceptual understanding: place value counters on grids

Signs and Missing Numbers



Mistakes / Misconceptions

Carrying digits incorrectly and using incorrect place value headings.

Concrete resources to support conceptual understanding: place value counters on grids

Signs and Missing numbers

2.4 ÷ 0.3 = ? x 1.25

Mistakes and Misconceptions

As previous years and BODMAS.

Fluency: 4x, 8x tables, 10 times bigger, 3x, 6x and 12x tables, Double larger numbers and decimals, 3x, 9x tables 11x, 7 x tables 6x, 12 x tables	**Fluency: 4x, 8x tables 100, 1000 times bigger, 3x, 6x and 12x tables 10, 100, 1000 times smaller Double larger numbers and decimals, 9x tables 11x , 7 x tables**	**Fluency: Multiplication facts up to 12 x 12** **Partition to multiply mentally** **Double larger numbers and decimals** **Multiplication facts up to 12 x 12**

Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Recognise and use factor pairs and commutativity in mental calculations	**Multiply and divide numbers mentally drawing upon known facts Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers establish whether a number up to 100 is prime**	**Perform mental calculations, including with mixed operations and large numbers**

Division

Foundation Stage

Begin to use the vocabulary of dividing, sharing and halving

Share objects into equal groups and count how many in each group

Pictorial
Share out a number of objects in to equal groups
There are 9 sweets. Can we share these equally between three friends?

Concrete resources to support conceptual understanding: **Numicon**
Manipulatives such as compare bears, unifix cubes, counters

Place value counters
Bead Strings
Dominoes
Dice

Bar Model

9		
3	3	3

Early learning goal – numbers

Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number. Using quantities and objects, they add and subtract two single-digit numbers and count on or back to find the answer. They solve problems, including doubling, halving and sharing.

Year One

Solve (practical) problems that involve sharing into equal groups

Pictorial

12 children get into teams of 4 to play a game. How many teams are there?

Bar Model

6	
3	3

6		
?	?	?

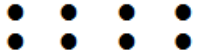
Year Two

Division as sharing and grouping (including remainders) 2 digit ÷ 1 digit (where divisor is 2, 5 or 10)

Pictorial: Understand division as sharing and grouping in groups or on a number line

Sharing – 6 sweets are shared between 2 people. How many do they have each?

8 ÷ 2 can be modelled as:



Grouping – There are 6 sweets. How many people can have 2 each? (How many 2's make 6?)

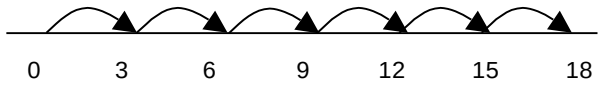
Year Three

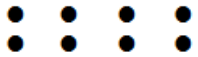
2 digit ÷ 1 digit (where divisor is 2, 3, 4, 5, 6 or 10)

Pictorial: Number line

Understand division as sharing and grouping
18 ÷ 3 can be modelled as:
Sharing – 18 shared between 3

Grouping - How many 3's make 18?





Using arrays to support calcualtions.

Signs and Missing numbers

10 / 5 = ?
? = 10 / 5
10 / 2 = ?
5 = 10 / ?

Concrete resources to support conceptual understanding: Numicon can be used to demonstrate sharing by using a 10 piece, for e.g. and seeing how many 2's fit over.
They can record this by drawing around the numicon.

Counters and unifix blocks can also be used and shared into rings and bead strings can be grouped.

Mistakes and Misconceptions

- Some pupils may interpret '3 multiplied by 4' as '4 groups/lots of 3' rather than '3 groups/lots of 4'
- Some pupils may try to give whole number answers for the half of an odd number – eg Half of 9 is 4 (or 5)
- Some pupils may not share equally when solving division problems – eg Divide 10 by 2: Answer 6 and 4

Bar Model

20				
4	4	4	4	4

20			
?	?	?	?

Concrete resources to support conceptual understanding: Numicon, Cuisinaire can be used to demonstrate sharing by using a 10 piece, for e.g. and seeing how many 2's fit over.

Counters and unifix blocks can also be used and shared into rings and bead strings can be grouped.

÷ = signs and missing numbers

6 ÷ 2 = □	□ = 6 ÷ 2
6 ÷ □ = 3	3 = 6 ÷ □
□ ÷ 2 = 3	3 = □ ÷ 2
□ ÷ ∇ = 3	3 = □ ÷ ∇

Mistakes and Misconceptions

- Some pupils may the see the times tables as a list of isolated, unconnected statements
- Some pupils may write statements such as 2 ÷ 8 = 4
- Some pupils may think that 30 is odd because '3' is odd

Bar Model

24		
8	8	8

24			
?	?	?	?

Concrete resources to support conceptual understanding: Cuisenaire on a number track

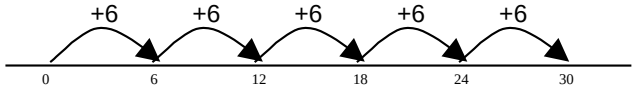
÷ = signs and missing numbers

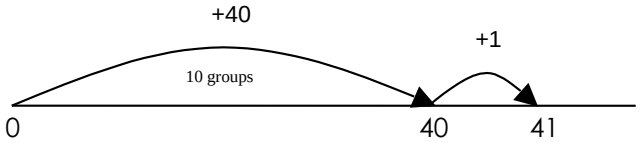
21 / 3 = ?
24 / ? = 8
3 = 33 / ?

Mistakes and Misconceptions

- Some pupils may write statements such as 2 ÷ 8 = 4
- Some pupils think because 3 × 5 = 5 × 3 then 15 ÷ 3 = 3 ÷ 15

Count back in 2s Count back in 10s Halves up to 10 Count back in 5s Halve multiples of 10 How many 2s? 5s? 10s?	**Fluency: Division facts (2 x table) Division facts (10 x table) Halves up to 20 Division facts (5 x table) Count back in 3s Review division facts (2x, 5x, 10x table)**	**Fluency: Review division facts (2x, 5x, 10xtable) Division facts (4 x table) Halve two digit numbers Division facts (8 x table) Division facts (3 x table) Division facts (6 x table) or review other**
Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher	**Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot. Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems**	**I can write and calculate mathematical statements for multiplication and division using the multiplication tables, including for 2-digit numbers, using mental and progressing to formal written methods.**

Division		
Year Four	**Year Five**	**Year Six**
Record, support and explain: 2 digit ÷ 1 digit (eg 98 ÷ 6) **Grouping and chunking** 30 ÷ 6 can be modelled as: grouping – groups of 6 taken away and the number of groups counted e.g.  **Chunking** using an empty number line – What can we multiply 4 by to get to 41? Chunk in familiar steps. 10 x 4 = 40 Remainder 1	**Refine and use efficient methods: 3/4 digit ÷ 2/1 digit** **Remainders** Quotients expressed as fractions or decimal fractions 61 ÷ 4 = 15 ¼ or 15.25 **Formal Written Efficient Method** **Formal Written Methods** Short division methods	**Use efficient methods: 4 digit ÷ 2 digit (eg 123 ÷7) 3/4 digit with ÷ 2 digit** **Remainders** Quotients expressed as fractions or decimal fractions 676 ÷ 8 = 84.5 **Formal Written Methods** 977 ÷ 36 is approximately 1000 ÷ 40 = 25 5834 ÷ 26 **Formal Written Methods** Continue with chunking and short division methods Move on to long division.



Abstract: chunking without a numberline
105 ÷5 = 21

(10 x 5 = 50 therefore double this to get closer to 105)
20 x 5 = 100
1 x 5 = 5
20 + 1 = 21

Bar Model

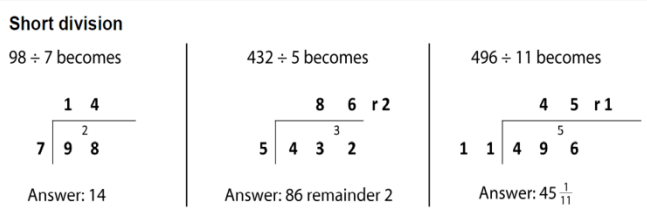
144											

63								

÷ = signs and missing numbers
12 = 144 / ?
81 / ? = 9
? / 6 = 7

Mistakes / Misconceptions
- Some pupils may write statements such as 2 ÷ 8 = 4

Fluency: Division facts (4x, 8x tables) 10 times smaller Division facts (3x, 6 x, 12x tables) Halve larger numbers and decimals Division facts (3x,

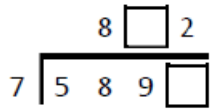


Bar Model

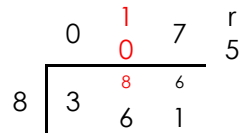
98						
14	14	14	14	14	14	14

98													
7	7	7	7	7	7	7	7	7	7	7	7	7	7

÷ = signs and missing numbers

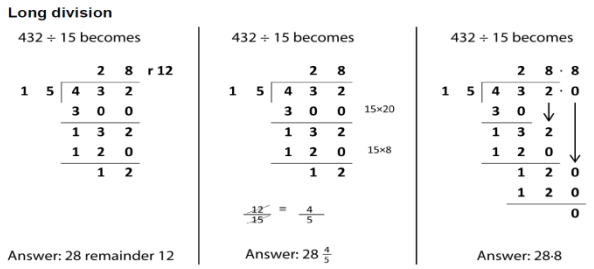


Mistakes and Misconceptions
- When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example:

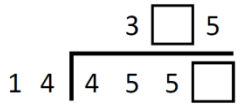


3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been 'moved across' and therefore everything that follows has been correctly carried out based on an early misunderstanding.

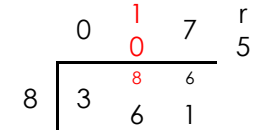
Fluency: Division facts (4x, 8x tables) 100, 1000 times smaller Division facts (3x, 6 x, 12x tables) Partition to divide mentally Halve larger numbers and decimals Division facts (3x, 9x tables) 100, 1000



÷ = signs and missing numbers



Mistakes and Misconceptions
- Some pupils may write statements such as 12 ÷ 132 = 11
- Formal written methods of addition, subtraction and multiplication work from right to left. Formal division works from left to right.
- When using short division many pupils will at first struggle to deal correctly with any division where the divisor is greater than the first digit of the dividend; for example:



3 ÷ 8 = 0 remainder 3, and so the 3 should be moved across. Instead, the 8 has been 'moved across' and therefore everything that follows has been correctly carried out based on an early misunderstanding.

Fluency: Division facts (up to 12 x 12) Partition to divide mentally Halve larger numbers and decimals Division facts (up to 12 x 12) Partition

9x tables) Division facts (11x, 7x tables) Division facts (6x, 12x tables)	**times smaller Review division facts (11x, 7x tables) Partition decimals to divide mentally Review division facts (6x, 12x tables) Halve larger numbers and decimals**	**to divide mentally. Halve larger numbers and decimals**
Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers Recognise and use factor pairs and commutativity in mental calculations	**Multiply and divide numbers mentally drawing upon known facts Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000**	**Perform mental calculations, including with mixed operations and large numbers with mixed operations and large numbers**